TOPOLOGY II MID-SEM EXAM

Time : 2 hours Max. Marks : 40

Answer all questions. You may use results proved in class after correctly quoting them. Any other claim must be accompanied by a proof. In this question paper $a, b \in \pi_1(S^1 \vee S^1)$ will denote the usual two free generators.

(1) Decide whether the following statements are *true* of *false*. Answers without correct and complete justifications will not be awarded any marks.

(a) If X is metric space and $A \subset X$ a closed subset homeomorphic to $\mathbb{R}$, then there exists a continuous function $f : X \longrightarrow X$ that has no fixed points.

(b) If B is connected and locally path connected and E, E' are two connected covers of B with

$$\mathrm{Cov}(E/B) \cong \mathrm{Cov}(E'/B),$$

then the two covering spaces are equivalent.

(c) Let $X = [0,1] \times [0,1]/\sim$ where $(0,x) \sim (1, 1-x)$ with the quotient topology. Then X is simply connected.

(d) Let X, Y be spaces that are homotopically equivalent. Then X is Hausdorff if and only if Y is Hausdorff.

(e) The subgroup $G \leq \pi_1(S^1 \vee S^1)$ generated by ab is normal. (No algebraic proofs please.)

[4xe=20]

(2) Construct a connected covering of $S^1 \vee S^1$ corresponding to the normal subgroup G of $\pi_1(S^1 \vee S^1)$ given by $G = << a^2, b^2, (ab)^4 >>$. Explain how the Deck transformation group acts transitively on the fibers. What can you say about the Deck transformation group?

[9+1+0]

(3) Let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be the linear transformation given by $T(x,y) = (2x, y/2)$. This generates an action of $\mathbb{Z}$ on $X = \mathbb{R}^2 - 0$. Then

(a) Prove that this is a covering space action.

(b) Show that $X/\mathbb{Z}$ is not Hausdorff.

(c) Compute $\pi_1(X/\mathbb{Z})$. [3+3+4]